

# CORRECTION AND ADDITION TO SOME THEOREMS CONCERNING PARTITIONS<sup>(1)</sup>

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Theorem 2 of the above mentioned paper is valid only provided that the set  $\{a\}$  of smallest positive residues does not consist of the single element  $a=q$ . Indeed, if it is required that all summands of a partition be divisible by the prime  $q$ , then, clearly,  $p_n(q) = p_n(q, l) = 0$  for  $n \not\equiv 0 \pmod{q}$ , while, for  $n = qn_1$ ,  $p_{qn_1} = p_{n_1}(1)$  and  $p_{qn_1} = p_{n_1}(1, l)$ . Here  $p_{n_1}(1)$  and  $p_{n_1}(1, l)$  are the corresponding partitions without congruence restrictions; they may be obtained from the formulae of Theorem 2, by setting formally  $m=q=1$ .  $p_n(1)$  is, of course, the number of unrestricted partitions and is well-known (see [2] and [10]). It is easy to see that this is actually the only case in which the statement of Theorem 2 needs a modification. Indeed, Theorem 2 is an immediate consequence of the lemma. The proof that conditions (b) and (c) of the lemma hold for the generating functions  $F(x)$  and  $H(x)$  does not depend on the set  $\{a\}$ . But the verification of condition (a) makes essential use of the fact that (see text on top of p. 124 and on p. 120, after (11))  $L/k^2 \leq t\Lambda$ , with  $t = \max_a B/q^2 = 1 - 6(q-1)/q^2 < 1$ . In case  $k=q=a$ , however,  $A=B=q^2$  and, if  $m=1$ ,  $\Lambda = \pi^2/6q$ ,  $L = \pi^2 q/6$  so that  $L/k^2 = L/q^2 = \Lambda > t\Lambda$ . This simply reflects the fact, evident from  $F_0(x) = \prod_{r=1}^{\infty} (1 - x^{q^r})^{-1}$ , that if  $x = \exp \{ \log r + 2\pi i h/q \}$ , then  $|F_0(x)|$  takes on the same value, for every integer  $h$ ; the situation for  $H(x)$  is similar. If, however,  $m > 1$ , then  $L = (\pi^2/6q) \sum_{a \in \{a\}} B \leq (\pi^2/6q) [(m-1)tq^2 + q^2] = \pi^2 m q t_1/6$ , with  $t_1 = [(m-1)t + 1]/m < 1$  and the argument of the test goes through with  $t_1 < 1$  instead of  $t$ .

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